Statistical process control for multistage manufacturing and service operations: a review and some extensions

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Abstract: Manufacturing and service processes today usually involve more than one process stages and operations. With an emphasis on achieving satisfactory product and service quality, multistage processes surveillance and fault diagnosis has become a necessity. Statistical Process Control (SPC) methods have been widely recognised as effective approaches for process monitoring and diagnosis. However, most conventional SPC methods focus on single-stage process without considering the multistage scenario. In this paper, we attempt to offer comprehensive references of statistical control methods for multistage manufacturing and service operations. Existing methods are compared and some future research topics are discussed.

Keywords: multistage manufacturing process; multistage service industry; Statistical Process Control; SPC; state space model.

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Introduction

As modern technology becomes increasingly sophisticated, most manufacturing industries, such as the Printed Circuit Board (PCB), semiconductor manufacturing, automotive body assembling, aerospace and many others include not just a single operation stage, but a large number of operation stages, namely multistage or multistations which we use interchangeably. For instance, Kim and May (1999) introduced a via formation procedure of multichip module dielectric layers made of photosensitive benzocyclobutene. The via formation process involves several sequential unique stages: spin coating, soft baking, exposing, developing, curing and plasma descuming. Likewise, the thriving demand for professional and meticulous service asks for more and more detailed labour division, services in telecommunication, banks and healthcare industries can be similarly viewed as a multistage process. For example, it may require the devotion of the sales clerks, cashiers and even the employees in the warehouse to provide customers shopping in a supermarket satisfactory service. For an international terminal, the vessel discharging process can also be divided into several stages: the quay cranes first unload the containers to the tractors, next the tractors transport the containers to the container yard and then the yard cranes unload the containers onto the stacks. In most cases, outputs from operations at downstream stages can be affected by operations at upstream stages. A product part or service transferring from one stage to the next stage in a multistage process may introduce extra variations that do not occur in a generic single-stage process. Thanks to recent advances in sensoring and information technology, automatic data acquisition techniques are commonly used in increasingly complicated processes with multiple stages, and a large amount of data and information related to quality measurement has become available. Thus, engineering and statistical approaches to make use of the multistage data and information regarding control and monitoring have become possible and beneficial in both industrial and service practices.

A large variety of Statistical Process Control (SPC) schemes have been developed for quality and productivity improvement since the 1960s. SPC utilises statistical methods to monitor manufacturing processes with an aim to maintain and improve the product quality while decreasing the variance. Much research has been conducted on the issues of SPC and the resulting developments are readily available in the literature, see surveys of research on SPC by Lowry and Montgomery (1995), Woodall and Montgomery (1999) and Stoumbos et al. (2000). Nevertheless, conventional SPC methods are typically restricted to a single process stage in industrial and service applications.

The last decades have witnessed great progress in studying multistage statistical process control. A body of literature can be found on the fault monitoring and identification in multistage operations quality surveillance. In this paper, we provide an extensive review of the literature on monitoring multistage manufacturing and service operations; the discussion also includes historical retrospection along with some ideas for future extension. The rest of this paper is organised as follows. In section 2 we discuss...
the development in modelling multistage manufacturing processes. Multistage service operations monitoring methods are introduced in Section 3. An overview of statistical process control methods for multistage processes is presented in Section 4. Section 5 contains extended methods for existing unsolved problems in multistage processes monitoring and diagnosis. The conclusion is presented in Section 6.

2 Multistage manufacturing operations

Many practical applications in multistage manufacturing processes can be found in recent literature. Zhou et al. (2003b) discussed an example of the 2D panel autobody assembly process, which contains multiple stages of assembly operations and product inspection for surface finish, joint quality and dimensional defects. The authors also reported another example in which hundreds of different stages were involved, and more than 30 stages were needed just for the engine-head machining. Djurdjanovic and Ni (2001) studied a machining process that is a combination of multiple machining operation stages. A key problem in monitoring a multistage process is how to describe such a process. Historically, most of the researchers tried to model the multistage process with statistical models, such as linear regression models. However, it is essential to incorporate engineering knowledge in multistage modelling and analysis for more effective process control and monitoring. Statistical model-based methods usually cannot describe the relationship among stages explicitly due to a lack of engineering background and knowledge. Thus, a large variety of current literature adopts multistage engineering models in a linear state space model structure based on physical laws and engineering knowledge that describe the quality information of a multistage process. See, for example, Jin and Shi (1999) and Ding et al. (2002b) for rigid-part assembly processes and Djurdjanovic and Ni (2001), Huang et al. (2002a,b) and Zhou et al. (2003b) for multistage machining processes. Also, Lawless et al. (1999) and Agrawal et al. (1999) discussed an AR (1) model, which could be put in a linear state space form, for representing the variation transmission in both multistage assembly and machining processes. The linear state space model structure provides an analytical engineering tool for modelling, analysing and diagnosing a multistage process. An extensive review of state space model can be found in Basseville and Nikiforov (1993) and Ding et al. (2002a). A book-length treatment of unified state space modelling methodology of multistage process can be found in Shi (2007).

The hierarchical nature of data generated from the multistage process suggests a two-level model. The first level involves the fitting of the quality measurements to the system input and quality information. The second level involves modelling the way that quality measurements change as a function of those measurements collected from previous process stages. Suppose a process has a total of \( N \) stages. Using the same notations as given by Jin and Shi (1999), a linear state space model of quality measurement on the \( k \)th stage of an in-control process is formulated by

\[
y_k = C_k x_k + w_k
\]

\[
x_k = A_{k,1} x_{k-1} + v_k
\]

For \( k = 1, K, N \), where \( x_k \) denotes the unobservable product quality information, such as the dimensional deviations of parts. The process noise, say common-cause variation
and unmodelled errors, is represented by \( v_k \), and the measurement error of the product quality is represented by \( w_k \). \( A_{k-1}x_{k-1} \) denotes the transformation of quality information from stage \( k-1 \) to stage \( k \). \( C_k \) is used to relate process states \( x_k \) to quality measures \( y_k \). \( A_k \) and \( C_k \) are known constant matrices at stage \( k \), which are derived or estimated from engineering knowledge, such as physical laws and process/product design information. Particularly, for univariate cases, \( v_k \) follows \( N(0, \sigma_v) \), \( w_k \) follows \( N(0, \sigma_w) \) with variance depending on stage index \( k \). The initial state \( x_0 \) follows \( N(\mu, \tau^2) \).

Process faults or out-of-control conditions could be fixture errors, machining errors, thermal errors, measurement errors, etc. As summarised by Basseville and Nikiforov (1993), possible faults in a state space model can be roughly classified into two types, additive change and non-additive change. Most of the multistage process monitoring methods emphasise on detecting additive changes. The study on multistage processes monitoring and fault detection methods for nonadditive changes is still scanty. Zhou et al (2004) used a linear state space model with a fault effect term, \( u_k \), added to the right-hand side of the second equation in model (1) to fit the possible faults in the state transition between successive stages. In particular, their work focused on the variation diagnosability problem with known parameters, where the true in-control process parameters were assumed to be known or could be accurately estimated. In Figure 1, we illustrate the quality information flow of a multistage process whenever the aforementioned fault occurs.

**Figure 1** Diagram of a multistage process

3 **Multistage service operations**

Many researchers have noticed the trend that service quality improvement has become a necessity in many industries. Wyckoff (1984) claimed that SPC is a good method for service managers to monitor service processes, and also helpful for staff to conduct self-improvement. Palm et al. (1997) also pointed out that SPC would have great possibilities in service industries, such as health care and education, and has already been proven to be useful in healthcare industry. The adoption of SPC into service operations provides a huge opportunity for service quality improvement. However, there are also some obstacles to applying SPC in services, such as what to measure and how to measure. The main difference between a manufacturing system and service system is that customers are involved into service operations. How to measure the customers’ perceived quality is a challenge. Therefore, researchers investigated modification of quality
SPC for multistage manufacturing and service operations

One of the most popular definitions was proposed by Parasuraman et al. (1985). Service quality was defined as the extent to which the service meets or exceeds customers’ expectations of it. Parasuraman et al. (1988) identified five service dimensions from their survey across industries and developed the service quality measurement scale called SERVQUAL, which has been a widely used measurement scale of service quality (Athanassopoulos, 1998; Lee et al., 2000; Soteriou and Chase; 1998; Soteriou and Zenios, 1999). TOPSIS (Hwang and Lin, 1987) and Loss Function (Ross, 1988) are also alternatives of perceptual quality measurement. Besides the perceptual measurement data of service quality, Klassen et al. (1998) identified the operating data of productivity, efficiency and effectiveness as three of the most widely used indicators in service operations.

Many service operations are in a series of stages, but research in services area usually assumed that there is no relationship between stages ((Souteriou and Hadjinicola, 1999) and Van Looy et al (1998)). Based on that, Souteriou and Hadjinicola (1999) discussed the resource allocation problem in multistage service operations. In their research, they suggested a model to determine the resource allocation with a minimum loss of perceptual quality. Armstrong (1995) also proposed a model for a medical clinic problem by using perceptual data and service wait. Both models assumed that the service quality indicators in each stage are independent. It is still an open research question on how to handle the more practical situation that multiple stages are intercorrelated.

Another important subject in service operations is to detect abnormal patterns and frauds in a service process. Abnormal patterns in data may be caused by various special causes in the process. In the past few decades, SPC has become the major method to detect and monitor manufacturing processes. Various control charts have been invented and used. Recently, a few researchers also tried to apply conventional control charts to service processes (Apte and Reynolds, 1995; Mehring, 1995; Sulek et al., 1995). These applications showed that control charts had a positive effect in helping service managers to find out special causes in processes. However, when managers are dealing with a multistage service process, they should apply conventional control charts with caution. Conventional control charts may obtain a satisfactory monitoring performance only if quality measurements in every stage are indeed independent. Nevertheless, if the service process has a cascade property, that is, there may be some relationships between the quality measurements of current stage and those from upstream stages, conventional charts may not be as efficient in correctly identifying faulty stages. To solve such a problem, some advanced control charts have been invented, and many of them were based on a regression model (Mandel, 1969; Zhang, 1985, 1989; Hawkins, 1991, 1993). The cause-selecting control chart by Zhang (1985, 1989, 1992) is proven to be a useful approach to monitor cascade processes. Sulek et al. (2005) used the cause-selecting chart in a real grocery store process and compared it to the traditional Shewhart chart. The grocery store was modelled as a two-stage cascade process. According to their results, the cause-selecting chart outperformed the Shewhart chart in signalling abnormal patterns. However, the adoption of SPC to service industries/processes is sometimes not straightforward. Jensen and Markland (1996) claimed that when people use SERVQUAL model as a measurement scale of service quality, univariate control charts may not be applicable. Because SERVQUAL model is made of five dimensions, if each dimension has a control chart, there will be too many out-of-control signals in all. Another drawback is that univariate control charts cannot show managers which customer has a poor quality perception of the entire service process. Jensen and
Markland (1996) have constructed a ‘quality perception control chart’ which is based on SERVQUAL modelled quality data. Besides, Hotelling $T^2$ control chart and principal component analysis could also be used to replace the univariate control chart in service processes.

4 SPC for multistage operations

Although it is important for the practitioners to ensure and improve product quality by identifying and detecting out-of-control occurrences in multistage processes, it is not straightforward to extend conventional single-stage SPC techniques to a situation with multiple and correlated stages, where the input of the current process stage is related to the outputs of early stages. Efficient and effective multistage process control and monitoring remain a challenge due to the increasing complexity of multistage processes. A finished product may have up to hundreds of quality characteristics, and in each separate manufacturing stage, there may also be more than hundred of intermediate variables, which include system state variables and quality variables. Monitoring all of these variables becomes a challenging task. Therefore, to design a multistage process monitoring and diagnosis scheme, an important issue that needs to be considered is what to monitor and where to monitor. Below we first review the conventional SPC methods for multistage process control and then discuss recent advances in multistage processes monitoring and diagnosis.

4.1 Multistage processes monitoring

To detect out-of-control occurrences in multistage processes, we may monitor the quality measurements of the product at the final stage using SPC techniques (Montgomery, 2001), such as the Shewhart, the Cumulative Sum (CUSUM) and the Exponentially Weighted Moving Average (EWMA) control charts for univariate quality measurement and Hotelling’s $T^2$, multivariate CUSUM and EWMA charts, etc., for multivariate cases. Nevertheless, these univariate and multivariate tools are designed for a single-stage process and cannot effectively identify the stage with root cause in a multistage process. Alternatively, we may also monitor quality measurements in individual stages by charting them separately. Hayter and Kwok-Leung (1994) established simultaneous confidence intervals for each variable mean in the multivariate quality control problem. Variables that fall out of the confidence interval are treated as errant variables. However, when the dimension becomes large, the detection power of their method becomes unsatisfactorily low. Moreover, ignorance of the fact that quality measurements at a certain stage are affected by the output quality from a preceding stage wastes much valuable information. This cascade property of multistage processes leads to challenging problems in statistical process monitoring and diagnosis. The regression adjustment approach, developed by Hawkins (1991, 1993), is often preferred by practical users when the process variables are related, but any subset of the variables can be impacted by an assignable cause without that effect being transferred to the others (Lowry and Montgomery, 1995). Due to its ability to tackle the cascade properties of a multistage process, Hawkins’ approach is applicable to multistage processes control problems (Shu et al., 2004a). In particular, Rao et al. (1996) address the application of Hawkins’ regression adjustment in multistage cases under a Bayesian statistical
framework. Shu et al. (2004b) investigated the run-length distribution of regression control charts in the presence of parameter uncertainty. However, since the quality measurements from different stages could be highly correlated, directly using the regression adjustment approach to monitor residuals from regression models may lead to misleading conclusions due to the collinearity. Alternatively, the co-linearity problem in regression adjustment approach can be partially alleviated by using the cause-selecting chart, see Zhang (1984, 1985, 1989, 1992) and Wade and Woodall (1993) for a review. Unlike the regression adjustment approach, the cause-selecting method includes only two adjacent stages in its regression model, and consequently leads to an easy identification of out-of-control stages. Current studies on the potential use of the cause-selecting chart for multistage processes can be found in Shu et al. (2003), Shu and Tsung (2003) and Shu et al. (2004b, 2005).

Similarly, Zantek, Wright and Plante (2002) used a simultaneous-equation model to represent statistical relationships between quality measurements from multiple stages in a process. Zantek et al. (2006) proposed describing the multistage process with regression models and monitoring the residuals of stages with simultaneous CUSUM charts.

4.2 Multistage processes diagnosis

A large majority of recent research on multistage processes diagnosis adopts the engineering-based state space model. In this section, recent progress in research and implementation of the state space model in multistage processes diagnosis is thoroughly summarised. The existing fault diagnosis methods are roughly grouped into two categories: pattern-matching methods and estimation-based methods.

In the pattern-matching methods, Ceglarek and Shi (1996) developed a pattern matching method for fixture fault diagnosis in an automotive body assembly process. Suppose one of the stages of the process experiences a malfunction, a consequence of such a change will be reflected in the final product or downstream intermediate products. Based on the historical data, an off-line diagnostic model that encompasses all possible fault patterns in the past can be built. Measurement data are collected online and analysed using some multivariate statistical methods to extract the fault feature patterns. Fault isolation can then be conducted by mapping the feature patterns of real production data with the predetermined fault patterns generated from the analytical model through principal component analysis. However, the diagnostics of this method require building an off-line diagnostic model, thus the possible fault patterns are restricted by the historical data. Ding et al. (2002a) also gave a fault monitoring and diagnosis method for detecting single faults in multistage processes with an application to an auto body assembly industry, fixture faults are identified by pattern recognition approach. Huang et al. (2002b) utilized a state space variation propagation model for multistage processes. A virtual machining concept was applied to identify faults between stages, and further used to determine the root cause of process faults.

In the estimation methods, the variances contributed by the process faults are estimated in the state space model. Apley and Shi (1998) used least square method to estimate the faults and its variance. Zhou et al. (2004) formulated the multistage fault identification problem as a problem of estimation and hypothesis testing of a general linear mixed model. They used a MLE method and provided a detailed experimental study to illustrate the effectiveness of the proposed methodology. Recently, Ding et al.
(2005) summarised and compared the existing variance estimation methods and provided guidelines for choosing appropriate method under different scenarios. Furthermore, instead of investigating fault diagnosis methods, Ding et al. (2002a) and Zhou et al. (2003a) identified the necessary conditions under which an estimation method is applicable. Ding, et al. (2002b) studied the diagnosability of a multistage process. Zhou, et al. (2003b) also investigated the diagnosability of a multistage process with an example of two-dimensional panel assembly process.

5 Extensions on multistage SPC based on engineering models

Phase I analysis of SPC aims to identify the in-control condition in a set of historical observations collected over time, so that in-control parameter values could be estimated. Up to now, only little work has been done on Phase I analysis of multistage processes. In the pioneering work of Zou et al. (2008), a popular multivariate change-point detection scheme is integrated with specific direction information based on a multistage state space model. From this integration, a multivariate change-point method was established for effectively testing and estimating a sustained shift in a fixed multistage process sample.

On the other hand, the main concern of Phase II analysis of multistage processes is to detect the deviation from the in-control parameters estimated in Phase I as soon as possible, and at the same time, make correct diagnosis whenever the control chart gives out a signal; various statistical methods can be applied either to the original observations or to the residuals of each stage. A possible approach for multistage processes monitoring was given by Xiang and Tsung (2008); the state space model was adopted to describe a complex multistage monitoring problem. They assumed that fault only occurs in one stage and applied a Group Exponential Weighted Moving Average (GEWMA) charts to the one-step ahead forecast errors of each stage. Also, the EM algorithm was applied to estimate the model parameters since the model parameters are usually unknown in practice. Zantek (2007) also used state space models for multistage processes. The original observations were treated as multivariate data and multivariate process control techniques were applied when multiple faults occurred.

Several extensions in the multistage SPC research that are based on engineering models are further discussed in the following sections.

5.1 Identifying multiple faults in multistage processes

Real world is complex, and multiple faults could occur simultaneously. How to detect and identify multiple faulty stages in multistage processes is still rarely explored. Most of the literatures mentioned thus far only consider single faults in a multistage process. Even multiple faults are considered, efficient online diagnosis methods were not provided. Zantek (2007) gave much attention to the monitoring aspect, but ignored how to identify the distinct faults. Lai (2000) proposed using multiple hypotheses testing methods to diagnose and identify the faulty variables in a multivariate distribution, which can be naturally adjusted for a multistage process. However, Lai’s method may be invalid when a multistage process involves a large number of stages. The reason is that, as the number of stages increases, the detection power of the multiple hypotheses testing methods that aim at controlling type I error rate would reduce dramatically and become
useless in practice. Nevertheless, thanks to the advance in modern statistics, new multistage processes control and diagnosis schemes for detecting multiple faults could be established based on more powerful multiple hypotheses testing methods.

A possible approach for multistage processes diagnosis is to formulate it into a multiple hypothesis problem. Most change detection algorithms, according to Basseville and Nikiforov (1993), are based on generating ‘residuals’ from the measurements that reflect the change in the process and then designing decision rules based on the residuals. Therefore, based on model (1), the standardised one-step ahead forecast error of \( y_{n,j} \) given \( y_{n,j-1} \), as denoted by Durbin and Koopman (2001), \( e_{n,j} \), can be calculated from the recursive method below:

\[
e_{n,j} = \frac{\nu_{n,j}}{V_n} \\
\nu_{n,j} = y_{n,j} - C_n u_{n,j} \\
u_{n,j} = A_n u_{n,j-1} + G_n \nu_{n-1,j} \\
V_n = C_n^2 W_n + \sigma_e^2 \\
W_n = A_n^2 W_{n-1} - A_n C_n W_n G_n + \sigma_e^2 \\
G_n = \frac{A_n C_n W_n}{V_n}
\]

(2)

where the initial values are \( \nu_{i,j} = y_{i,j} - C_i A_i a_0 \), \( e_{n,j} = \nu_{n,j} / V_n^{1/2} \), \( u_{i,j} = 0 \) and \( W_j = \sigma_e^2 + A_j^2 \). When the process is in control, Durbin and Koopman (2001) showed that \( e_{n,j} \) is independently and identically distributed as a standard normal distribution, \( N(0,1) \). When a particular stage of the multistage process undergoes a step change in its state transition equation, the residual mean of that stage will experience a mean shift while its variance remains the same. Thus, the multistage monitoring and diagnosis may be formulated into a multiple hypotheses testing problem:

\[
H_{0,n} : e_{n,j} \sim N(0,1), n = 1, K , N \\
H_{i,n} : e_{n,j} \sim N(\mu_n,1), n = 1, K , N
\]

(3)

where \( \mu_n \neq 0 \).

Conventional multiple hypotheses testing methods applied to multistage processes monitoring are mostly Bonferroni-type methods, such as in Zantek et al. (2006). However, the power of such testing methods aiming at controlling the type I error rate becomes unsatisfactorily low when the number of hypotheses is large. A similar problem of high-dimensional multiple hypotheses testing also occurs in microarray genetics research. Microarray data may consist of up to thousands of individual DNA sequences. To compare gene expression between the control and treatment groups, researchers have to simultaneously test up to thousands of hypotheses. When conventional multiple hypotheses testing methods are used to differentiate genes, their power becomes so low that only few different pairs can be found. To tackle this problem, a False Discovery Rate (FDR) control approach has been proposed (Benjamini and Hochberg, 1995) and used extensively in recent microarray research. Instead of
controlling the probability of making one single wrong rejection, FDR approach controls
the proportion of wrong rejections to the number of all the rejected hypotheses. It has
been proved that when all the hypotheses are exactly true, controlling the FDR is
equivalent to controlling type I error rate. When some of the alternative hypotheses are
true, as in reality, controlling the FDR would provide much higher power. Li and Tsung
(2008) introduced FDR control method into multistage processes monitoring and
diagnosis. By integrating the false discovery rate control method and the conventional
control schemes together, the simultaneous Shewhart, CUSUM or EWMA control charts
proposed by Li and Tsung (2008) demonstrated higher multistage processes monitoring
and diagnosis capability.

5.2 Multistage processes chart allocation

Chart allocation problem in multistage processes is another important issue that needs to
be emphasized. Cost and resources are always limited in reality and it is not always
possible to measure outputs and set up traditional control chart in every stage in a
multistage process. Previously, chart allocation decisions are usually made based on
common sense and experience. Due to the lack of systematic way to support the
decisions, many of the decisions lead to deteriorated monitoring performance and wasted
resources. Inspired by the cascade properties embedded in multistage processes, Jin and
Tsung (2008) provided a more systematic and economic strategy for control charts
allocation. Provided that stage correlation is treated as a critical factor
in determining the decisions, efficiently monitoring the multistage process with simple
control charts becomes possible. Agrawal, Lawless and MacKay (1999) analyzed
the variation propagation model, which is in the form of a state space model, of the
multistage process, with an aim to identify the stages which contribute most
to the variance of the final product. Their results provide a clue to which stage should be
monitored. Therefore, by allocating control charts in appropriate position in the
multistage process, better performance in monitoring multistage processes can be
achieved. Up to now, not much research works have been done on this topic.

If we can identify fault patterns of mean shift offline using the modern techniques,
such as CAD, or expert knowledge (Ceglarek and Shi, 1996, Apley and Shi, 1998,
Ceglarek et al. 1994), an appropriate chart allocation strategy could be possibly designed:
According to the stream-of-variance (SOV) model in a state space form (see equation
(1)) (Shi, 2007), the correlations between stages and the propagation of faults can be
obtained. By using the state space model, the noncentrality parameter- $\delta$, in the
downstream stages of the fault, can be obtained as:

$$\delta = \frac{C_{i} \left( \prod_{k=i}^n \sigma_i A_k U_i \right)}{\sqrt{\sum_{i=1}^n \left[ C_{i} A_{i-1} \ldots A_1 \right] ^2 \sigma^2 + C_{i}^2 \sigma^2 + \sigma^2}}$$

(4)

Based on the above result and the type of control chart we use, say, EWMA chart, we
can obtain the out-of-control ARL of the downstream stages of the fault (Prabhu and
Runger, 1997). If we consider processing delay between stages, the Average Time to
Signal (ATS) can also be obtained:

$$ATS = ARL \times t + \text{delay}$$

(5)
where $t$ is the time interval between runs. The stage which has the smallest ATS is the appropriate stage for the fault. The above approach can also be extended to multivariate cases with matrix manipulations.

6 Conclusion

Rapid development of modern technology, together with advances in information, sensing and data acquisition technology, provide large volumes of data that are routinely collected in a multistage environment. To effectively utilise such data and develop effective process control methodologies for multistage manufacturing and service operations is a very challenging task. In this paper, a review of SPC methods for manufacturing and service industries is provided. Various statistical methods can then be applied to either original quality measurements or residuals. In particular, the engineering-based state space model is reviewed and recommended to characterize multistage processes. However, research on multistage processes monitoring and diagnosis is far from adequate. Technical challenges still remain. For example, although the state space model can be used to describe a multistage service process as in a manufacturing process, how to determine the transition matrix and variances in the state space model in service processes is still a challenge.

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